

Lecture 8: Two forces - Gravity and Friction

Many of the early experiments in mechanics were directed at disentangling the roles of gravity and friction. Today we are going to first talk more about gravity, then about friction and finally about problems where both gravity and friction are important. Note that these laws are based on experiment, as all the laws of physics are.

Gravity and weight

In physics, the weight of an object is $w = mg$ and it is due to the force of gravity. The mks unit for weight (and the unit of any force) is the Newton $N = kgm/s^2$. On the moon the weight of a person is much smaller than on earth, while on Jupiter a person's weight is much larger. In general all massive objects attract each other through the "Universal law of gravity" also discovered by Newton and discussed in Principia. He said that two spherical objects with masses m and M attract each other with a force given by,

$$F = \frac{GMm}{r^2} = mg \quad (1)$$

where r is the distance between the centers of the two masses and G is a constant $G = 6.67 * 10^{-11} Nm^2/kg^2$. From this equation we see that the gravitational acceleration at a distance r from the center of a spherical mass M is,

$$g = \frac{GM}{r^2}. \quad (2)$$

Using this expression we calculate the acceleration due to gravity at the surface of the earth, using data for the mass and radius of the earth,

$$g = \frac{6.67 * 10^{-11} \frac{Nm^2}{kg^2} * 5.98 * 10^{24} kg}{(6.38 * 10^6 m)^2} = 9.8m/s^2 \quad (3)$$

Note that g actually varies a little bit from place to place on the surface of the earth due to variations in density, deviations from spherical shape and elevation. These variations can be used in oil and gold exploration from the air.

From Eq. (2) it is evident that as the distance from the center of the earth increases, the acceleration due to gravity decreases. We actually weigh

a little bit less on top of mount Everest than we do at sea level, though our mass is always the same. It is also evident from Eq. (2) that the gravitational attraction increases linearly with the mass of a planet. Small mass planets have weak gravity, but large mass planets have large gravity. The highest gravity occurs for small objects that have really large mass, like white dwarf stars, neutron stars and the ultimate massive object, a large black hole.

Friction

Friction is a force that resists motion. It never initiates motion. It is a force which opposes a net external force which tries to produce motion.

The force of friction is different when a body is moving than when it is stationary, so we have to define two friction coefficients: The coefficient of static friction μ_s and the coefficient of kinetic friction μ_k . The magnitude of the force due to friction is given by

$$F_f = \mu N \quad (4)$$

where $N = mg$ is the weight and μ is dimensionless and is equal to μ_s when the object is stationary and it is equal to μ_k when the object is moving. The friction law is really surprising for the following reasons:

- (i) It does not depend on the shape of the object
- (ii) The kinetic friction force does not depend on the velocity of the object.

Note that the kinetic friction force is always $\mu_k N$, however the static friction force is equal to the applied force up to the value $\mu_s N$. An external force, F , less than $\mu_s N$ cannot move the object, while forces larger than $\mu_s N$ lead to motion and then we use the kinetic friction law to find that the net force producing acceleration is $F - \mu_k N$.

Example: Friction, gravity and motion rolled up together

Consider a block on an inclined plane where the static friction coefficient is $\mu_s = 0.4$ and the dynamic friction coefficient is $\mu_k = 0.25$. Imagine slowly increasing the angle of the inclined plane until motion of the block just starts. Find the angle at which motion commences and calculate the velocity and displacement of the block 5s after it begins to slide.

Solution

If θ is the angle of the inclined plane, then the normal force of the block on the inclined plane is $N = mg \cos(\theta)$. The component of the block's weight

down the inclined plane is $mg\text{Sin}(\theta)$. The maximum force that static friction can resist is $\mu_s N = \mu_s mg\text{Cos}(\theta)$. The critical angle, θ_c , for motion to start is then found from,

$$\mu_s mg\text{Cos}(\theta_c) = mg\text{Sin}(\theta_c), \quad (5)$$

from which we find that the critical angle is given by,

$$\theta_c = \text{ArcTan}(\mu_s) = 22^\circ. \quad (6)$$

Once motion has started the force along the inclined plane is given by,

$$\sum F_{\text{inclined plane}} = mg(\text{Sin}(\theta) - \mu_k \text{Cos}(\theta)) \quad (7)$$

so the acceleration along the inclined plane is

$$a = \sum F/m = g(\text{Sin}(22) - 0.25\text{Cos}(22)) = 1.37\text{m/s}^2. \quad (8)$$

This is motion at constant acceleration along the inclined plane, so the velocity after 5s is given by,

$$v = v_0 + at = 1.37 * 5 = 6.83\text{m/s} \quad (\text{down inclined plane}) \quad (9)$$

and the displacement is,

$$\Delta x = v_0 t + at^2/2 = 17.1\text{m} \quad (\text{down inclined plane}) \quad (10)$$